Currency Competition with Uncertain Acceptance: The Case of Cryptocurrencies*

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Abstract

Prices of new currencies reflect both the liquidity of a currency today and its expected future liquidity as more consumers and merchants accept it as means of payment. We adopt a New Monetarist framework with an emerging currency, cryptocurrency, competing with an existing currency, money. Seller's acceptance of cryptocurrency exogenously grows over time; this growth stops in a random period. Cryptocurrency prices increase in expected future acceptance and crash when acceptance growth stops. We also study an environment in which a fraction of buyers are optimistic about the future of cryptocurrencies. Surprisingly, the presence of optimists has an ambiguous effect on prices.

Keywords: Money, Search, Private Money

JEL codes: E40, E41

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1 Introduction

Since the creation of Bitcoin by Satoshi Nakamoto in 2008, cryptocurrencies, cash-like electronic payment systems issued by private entities, have substantially grown in popularity.[1] Users are drawn to these currencies partly for transaction motives – low transaction fees (especially for international transactions), fixed currency supply schedules, and anonymous/pseudonymous transactions – but also for speculative reasons. Because most cryptocurrencies are fiat assets, their long run viability depends on their transaction (liquidity) value; this paper focuses on transaction motives. Specifically, we study how current seller acceptance and beliefs about future firm acceptance affect cryptocurrency prices. When sellers start accepting these currencies as means of payment, the transaction value of cryptocurrencies increase and prior beliefs about the usefulness of cryptocurrency as a means of payment are (partially) justified.[2] However, seller adoption takes time, some sellers are willing to accept cryptocurrency sooner than others. Because cryptocurrency prices depend on potential future acceptance, events that hurt beliefs about the growth of seller acceptance, such as China's outlawing of Bitcoin exchanges or the US's disapproval of Facebook's proposed cryptocurrency Libra, can have large effects on prices even if they have small effects on current acceptance.

This paper answers the following questions: How do beliefs about cryptocurrency acceptance affect prices? How important is expected acceptance compared to current acceptance? When do people adopt or abandon cryptocurrency? How does the presence of cryptocurrency affect welfare? How do heterogeneous beliefs about future acceptance affect current prices?

Our paper examines competition between two currencies: an emerging currency, cryptocurrency, and an existing currency, money, that differ in acceptance rate and currency

[1] An overview of what cryptocurrencies are and their benefits and downsides can be found in Böhme et al. (2015) or Berentsen et al. (2018). Dwyer (2015) gives an overview of virtual currency with a focus on Bitcoin.

[2] One example of cryptocurrency used to buy goods is when KFC Canada introduced a "Bitcoin Bucket" of fried chicken and waffle fries in 2018. A short list of companies that accept cryptocurrencies as payments include tech companies like Microsoft, but also non-tech firms such as Expedia and Subway.

stock growth rate. Money is universally accepted but is inferior at preserving value because its stock grows at a faster rate. Currently, cryptocurrency competes with some local currencies, like Venezuela Bolivar, that are experiencing hyperinflation.[3] To study the liquidity value of these currencies, we adopt a Lagos and Wright (2005) (LW) framework where assets are used to overcome trading frictions.[4] Our main contribution is to study how changes in future currency acceptance affect prices today.

Inspired by Zeira (1999), we add informational dynamics to study cryptocurrency markets' booms and crashes. Cryptocurrency is accepted by a fraction of sellers, but this fraction exogenously increases over time.[5] Agents expect the acceptance rate of cryptocurrency to follow a known growth path until a random termination date, after which cryptocurrency acceptance will stay constant forever. Upon reaching the termination date, the world enters steady state and real balances stay constant; the environment is the same as a standard LW. Each period, after choosing their currency portfolios, agents observe the acceptance rate and update their beliefs about the termination date. Because today's cryptocurrency price depends on tomorrow's price, while acceptance rate is growing, cryptocurrency price exceeds steady state value. However, when agents realize that acceptance growth had stopped, the price of cryptocurrency immediately crashes to its steady state value. Like Zeira, cryptocurrency crashes occur even if buyers are rational and have correct beliefs about the termination date. Unlike Zeira, our assets are intrinsically valueless, so prices can potentially crash to zero if not enough sellers accept cryptocurrency.

Because our economy has a random finite termination date, we can use backwards induction to find a well-defined solution for prices and quantities traded each period. We find that the potential for growth in cryptocurrency acceptance raises expected future prices, which in

[3]For more on usage of cryptocurrencies in Venezuela, see `https://www.nytimes.com/2019/02/23/opinion/sunday/venezuela-bitcoin-inflation-cryptocurrencies.html`.

[4]Or, more accurately, a Rocheteau and Wright (2005) framework, as we have permanent buyers and sellers. For more on the New Monetarist literature, see Lagos et al. (2017) and Williamson and Wright (2010).

[5]Appendix D shows a model with endogenous cryptocurrency acceptance by sellers choose to begin accepting cryptocurrency.

turn lowers expected cryptocurrency inflation. Lower expected inflation means higher cryptocurrency prices today and makes holding cryptocurrency worthwhile for buyers even when acceptance is low and cryptocurrency would not be valued if acceptance growth was stagnant. This means cryptocurrency growth have a large effect on prices even when current acceptance is low. Higher demand for cryptocurrency crowds out demand for money, so money prices are below steady state levels and decrease in cryptocurrency acceptance. If buyers are more optimistic about future acceptance, cryptocurrency prices increase and money prices decrease. Once acceptance stops growing, cryptocurrency prices fall and money prices jump to steady state values. If acceptance is too low, buyers will abandon cryptocurrency. We also study total welfare in the setting of competing currencies. Total welfare is determined by trade between buyers and sellers which is increasing and concave in buyer's real balances. Because of this, agents are always worse off with higher money growth rate but may be better or worse off with higher cryptocurrency growth rate or acceptance rate. Intuitively, the presence of cryptocurrency crowds out money holdings, which can make buyers much worse off when they meet sellers who only accept money.

People have heterogeneous beliefs about future cryptocurrency acceptance and start adopting cryptocurrencies at different times.[6] We extend the model to parsimoniously capture this heterogeneity. In the extended model, a fraction of buyers we call normal buyers behave as in the baseline model, the rest are optimistic buyers we call hodlers who believe that acceptance of cryptocurrency will always grow.[7] While simplistic, this framework sheds light on how early adopters affect prices as well as cryptocurrency holdings of normal buyers. Hodlers' optimism causes them to hold more cryptocurrency and less money than normal buyers. At low levels of acceptance, they can crowd normal buyers out of the cryptocurrency market, but when cryptocurrency is accepted by enough sellers, normal buyers will start to use cryptocurrency. Interestingly, adding hodlers to the baseline model has an ambiguous

[6] Athey et al. (2016) estimates that there are 27,474,538 entities using Bitcoin as of Nov. 2015. This is a reasonable upper bound of the number of worldwide Bitcoin users.

[7] The word hodler, a corruption of the word holder, has come to stand for "Hold-On-for-Dear-Life-ER" or people who believe Bitcoin will keep rising in price.

effect on prices. The presence of hodlers can increase cryptocurrency prices because they are optimistic about its future and value it more today. But the price of cryptocurrency can be lower because currencies are valued for their marginal liquidity, and the marginal liquidity can be lower for both types compared to the baseline case. Hodlers may have large enough cryptocurrency real balances such that the marginal unit of cryptocurrency is worth less to them than to normal buyers in the baseline case. Normal buyers now replace some cryptocurrency holdings with money, which also decreases the value of the marginal unit of cryptocurrency compared to the baseline case. If both these effects are strong enough, cryptocurrency prices are lower.

Our model studies uncertain growth of cryptocurrency acceptance and the potential for crashes in the price of cryptocurrency. In this way, our paper is similar to Choi and Rocheteau (2019) which studies agents' decision to mine cryptocurrency or participate in trade. They also generate booms and crashes in prices, but their crashes can occur even with perfect foresight and prices that always crash to zero.[8] We abstract from the mining process and our crash relies on informal dynamics but post-crash cryptocurrency is still useful and prices can be positive.

Our information dynamics are inspired by Zeira (1999) who models crashes in asset prices as the result of uncertain termination of economic growth, such as the end of productivity increases or population booms. We apply these dynamics to fiat rather than real assets, which are valued for their liquidity rather than their dividends. Because the value of fiat assets is mainly determined by beliefs, incomplete information has a greater effect on prices. For example, our prices can start positive then crash to zero when acceptance growth stops, which can never happen with real assets.

While this paper is motivated by cryptocurrency, our model applies just as well to any pair of competing fiat currencies, i.e. Mexican Pesos and US Dollars. As such, our paper relates to the dual currency literature. The paper closest to ours is Lester et al. (2012), who also

[8] Other papers study cryptocurrency prices and crashes but do not consider liquidity: Glaser et al. (2014), Cheah and Fry (2015), Weber (2013), Cheung et al. (2015), and Donier and Bouchaud (2015).

study how an asset with more liquidity competes with an asset with higher returns, but they focus on endogenous acceptance of the higher return asset. Similarly, Zhang (2014) studies competing national currencies with endogenous acceptance of the other nation's currency. Both papers only study steady state equilibrium while we focus on dynamics where acceptance changes gradually over time to show how uncertainty of acceptance affects price and quantity traded. While we can endogenize acceptance in ways similar to these papers (see Appendix D), we focus on exogenous acceptance in the main body of the paper. Fernández-Villaverde and Sanches (2016) endogenize cryptocurrency creation by allowing entrepreneurs to create their own monies, while we take cryptocurrency issuance as exogenous. Schilling and Uhlig (2019) study an endowment economy with both money and cryptocurrency where acceptance is universal but with uncertain endowment, while our model addresses uncertain acceptance. Like Rocheteau and Wright (2013), we study how prices can change with expectations about future liquidity. Our model relies on seller acceptance of cryptocurrency changing over time while their model relies on changing beliefs about currency value.

The rest of paper is organized as follows. Section 2 overviews the baseline model: defining the model and equilibrium, examining steady state and comparative statics, introducing dynamics, and showing a numerical example. Section 3 shows the hodlers extension along with a numerical example and comparison to the baseline model. Section 4 concludes.

2 The Model

In this section, we describe the setup of our model and define equilibrium. We first derive steady state equilibrium conditions and show some useful comparative statics. Then we show how we recursively solve the model outside steady state. Finally, we show some numerical examples of the model to demonstrate booms and crashes.

2.1 The Environment

Our model builds on the money search framework of LW. Time is infinite and discrete. Each period has two subperiods: a frictional decentralized market (DM) in the first subperiod and a frictionless centralized market (CM) in the second subperiod. In the CM, all agents receive lump-sum transfers, produce and consume a numeraire good, and choose their optimal portfolio of money and cryptocurrency. In the DM, buyers and sellers meet bilaterally and bargain over price and quantity if matched. Both CM and DM goods are non-storable. We also assume anonymity in the DM to rule out the use of credit in trade. There are two types of fiat assets: government issued fiat money m and privately issued cryptocurrency c. Money supply M has growth rate $1 + \gamma_m = \frac{M_{t+1}}{M_t}$ and cryptocurrency supply C has growth rate $1 + \gamma_c = \frac{C_{t+1}}{C_t}$.

There is a unit measure of buyers and sellers who live forever. There are two types of sellers: crypto sellers have measure α and accept both money and cryptocurrency while money sellers have measure $1 - \alpha$ and only accept money. The measure of crypto and money sellers evolves over time. After each CM, some money sellers are exogenously provided the technology to recognize cryptocurrency and thus become crypto sellers.[9] The measure of crypto sellers evolves following $\alpha_{t+1} = g(\alpha_t)$ where $g'(\cdot) > 1$ so $\alpha_{t+1} > \alpha_t$. After some unknown period $T \in \{0, 1, ..., \overline{T}\}$, the measure of crypto sellers stays constant and we assume the world reaches a monetary steady state with constant real balances, where $\overline{T} < \infty$ is the largest possible termination date.[10] The law of motion of α is

$$\alpha_t = \begin{cases} g(\alpha_{t-1}) & \text{if } t < T; \\ \alpha_T & \text{if } t \geq T. \end{cases}$$

[9]As shown in Lester et al. (2012), endogenous acceptance allows multiple levels of seller acceptance due to different levels of coordination. Because we focus on dynamics, agents would need to form beliefs about future seller coordination. By keeping acceptance growth exogenous, we abstract from this issue. For a version of the model with endogenous acceptance, see Appendix D.

[10]As we show in Section 2.4, guaranteeing a termination date allows us to use backwards induction to solve for prices and quantities traded.

Agents discover that growth stopped in period T when they reach the DM of period $T+1$ and learn α did not grow. Buyers have a common prior over date $F(T)$ with pdf $f(t)$. We use π_t to denote buyers' beliefs exiting the CM about α staying constant conditional on α's growth history. If $t > T$, buyers know that acceptance growth has stopped and $\pi_t = 1$. If $t \leq T$, buyers know that acceptance grew last period but are unsure whether it will grow this period, so

$$\pi_t \equiv \mathbb{P}(\alpha_{t+1} = \alpha_t \,|\, \alpha_t > \alpha_{t-1}) = \mathbb{P}(t = T \,|\, t \leq T) = \frac{f(t)}{1 - F(t-1)}.$$

Because time is discrete, beliefs are not continuous. Define buyer's prior as $F(T) = \{\hat{\pi}_0, \hat{\pi}_1, ..., \hat{\pi}_{\overline{T}}\}$ where $\hat{\pi}_t \equiv \mathbb{P}(t = T)$. Then

$$\pi_t = \begin{cases} \frac{\hat{\pi}_t}{1 - \sum\limits_{\tau < t} \hat{\pi}_\tau} & \text{if } t \leq T \\ 1 & \text{if } t > T. \end{cases} \tag{1}$$

For now, we assume all agents have the same beliefs, an assumption we relax in Section 3.

The prices of money and cryptocurrency are denoted as ϕ and ψ respectively. In equilibrium, prices depend on currency stock, which only depends on time period t, acceptance rate α, and beliefs π; $\phi = \phi_t(\alpha, \pi)$ and $\psi = \psi_t(\alpha, \pi)$. Throughout the paper, we abuse notation and denote these as $\phi_{t,\alpha}$ and $\psi_{t,\alpha}$ while acceptance is still growing and $\bar{\phi}_{t,\alpha}$ and $\bar{\psi}_{t,\alpha}$ when acceptance has stopped growing and the economy is in steady state. When referring to individual real balances, we use $\omega = \phi m$ and $\upsilon = \psi c$, and we use effective wealth to denote the total real balances recognized by a seller, i.e. $\omega + \upsilon$ for crypto sellers and ω for money sellers.

2.2 Buyer's Problem

In the CM, a buyer facing acceptance rate α with initial portfolio (m, c) and belief π solves the following maximization problem

$$W(\omega, \upsilon, \alpha, \pi) = \max_{x,h,m',c'} \left\{ x - s + \beta \mathbb{E}_\pi \left[V(\omega', \upsilon', \alpha', \pi') \right] \right\}$$

$$\text{s.t. } x + \phi_\alpha m' + \psi_\alpha c' \leq s + \phi_\alpha m + \psi_\alpha c + \Omega^m + \Omega^c;$$

$$x \geq 0; \;\; m' \geq 0; \;\; c' \geq 0.$$

Buyers have linear utility of consumption x and disutility of working s.[11] Ω^m and Ω^c are the lump-sum transfer of currency by the government and private issuer respectively. At the end of the CM, buyers choose their asset portfolio for tomorrow (m', c') to maximize the continuation value V in the upcoming DM. Agents cannot short assets, so portfolio holdings are non-negative. Expectations about α' are taken over their current prior π and beliefs are updated as explained above.

In the DM, a buyer is matched with a seller with probability λ. Conditional on meeting a seller, the probability of meeting a crypto type seller is α while the probability of meeting a money type seller is $1 - \alpha$. The value of the DM is

$$V(\omega, \upsilon, \alpha, \pi) = \lambda \left\{ \alpha V^{cs}(\omega, \upsilon, \alpha, \pi) + (1 - \alpha) V^{ms}(\omega, \upsilon, \alpha, \pi) \right\} + (1 - \lambda) W(\omega, \upsilon, \alpha, \pi)$$

where V^j is the value of a buyer meeting a crypto or money seller $j \in \{cs, ms\}$. When a buyer meets with a seller, the terms of trade specify quantity q^j and payment d^j such that

[11] Wages from working are normalized to 1.

the buyer receives fraction θ of total surplus, i.e Kalai bargaining. They solve

$$V^j(\omega, \upsilon, \alpha, \pi) = \max_{q^j, d^j} \left\{ u(q^j) - d^j \right\} + W(\omega, \upsilon, \alpha, \pi) \tag{2}$$
$$s.t.\ \ u(q^j) - d^j = \frac{\theta}{1-\theta} \big[d^j - h(q^j) \big];$$
$$q^j \geq 0;\ \ d^j \leq \begin{cases} \omega + \upsilon & \text{if } j = cs, \\ \omega & \text{if } j = ms. \end{cases}$$

Quantity of exchange has to be non-negative and payment is constrained by effective wealth. Buyers' utility of consumption and sellers' disutility of production have the usual properties: $u' > 0,\ u'' < 0,\ u'(0) = \infty,\ h' > 0,\ h'' \geq 0$, and $h(0) = 0$.

We now define a few terms to simplify the problem. The efficient quantity traded in the DM is q^* which satisfies $u'(q^*) = h'(q^*)$. Suppose a buyer has effective wealth w. Let $q(w)$ be the quantity traded given the buyer's effective wealth, which solves Problem (2), and $S(w)$ be the total trading surplus in the DM given effective wealth w, i.e. $S(w) \equiv u[q(w)] - h[q(w)]$. Finally, we define $\ell(w)$ as the liquidity premium which corresponds to the buyer's marginal gain of carrying one additional unit of effective wealth w into a DM meeting, i.e.

$$\ell(w) \equiv \theta S'(w) = \begin{cases} \theta \frac{u'[q(w)] - h'[q(w)]}{z'[q(w)]} & \text{if } q(w) < q^*; \\ 0 & \text{otherwise}, \end{cases}$$

where $z[q(w)] \equiv (1-\theta)u[q(w)] + \theta h[q(w)]$ is the transfer of wealth from buyers to sellers. By bringing more effective wealth into a match, buyers can increase their trading surplus by raising the total surplus. However, if a buyer can already purchase the efficient amount, the extra wealth is unspent and carried into the next CM. The opportunity cost of holding money and cryptocurrency can be defined as $i^m_\alpha = \frac{\phi_\alpha}{\beta \mathbb{E}_\pi[\phi'_{\alpha'}]} - 1$ and $i^c_\alpha = \frac{\psi_\alpha}{\beta \mathbb{E}_\pi[\psi'_{\alpha'}]} - 1$. Following LW, the assumption of linear utility in the CM makes W linear in (m', c'). Using these definitions, buyers' CM maximization problem becomes

$$\max_{m',c'} \Big\{ -i_\alpha^m \mathbb{E}_\pi \phi'_{\alpha'} m' - i_\alpha^c \mathbb{E}_\pi \psi'_{\alpha'} c' + \lambda\theta \mathbb{E}_\pi \big[\alpha S(\phi'_{\alpha'} m' + \psi'_{\alpha'} c') + (1-\alpha) S(\phi'_{\alpha'} m') \big] \Big\}. \tag{3}$$

The sellers' problem can be similarly defined. We assume sellers have the same beliefs as buyers. Because sellers do not consume in the DM, they have no incentive to carry a positive amount of real balances into the trade meeting. Sellers consume all their wealth and choose portfolio $(m', c') = (0, 0)$ in the CM and split $(1-\theta)$ of the total surplus in the DM. Hence, a sequential monetary equilibrium can be defined as follows

Definition 2.1. Let $\boldsymbol{q} = (q^{cs}, q^{ms})$ and $\boldsymbol{w} = (\omega, \upsilon)$. Given initial prior $\{\hat{\pi}_t\}_{t=1}^{\overline{T}}$, define a *sequential monetary equilibrium* as a set of quantities traded
$\big\{\{\boldsymbol{q}_t(\pi_t, \alpha_t)\}_{t\leq T},\ \{\boldsymbol{q}_t(\pi_t, \alpha_T)\}_{t=T+1},\ \{\boldsymbol{q}_t(1, \alpha_T)\}_{t>T+1}\big\}_{t=0,\ T=0}^{\infty\quad \overline{T}}$ and real balances
$\big\{\{\boldsymbol{w}_t(\pi_t, \alpha_t)\}_{t\leq T},\ \{\boldsymbol{w}_t(\pi_t, \alpha_T)\}_{t=T+1},\ \{\boldsymbol{w}_t(1, \alpha_T)\}_{t>T+1}\big\}_{t=0,\ T=0}^{\infty\quad \overline{T}}$ such that[12]

1. $\big\{\{\boldsymbol{q}_t(\pi_t, \alpha_t)\}_{t\leq T},\ \{\boldsymbol{q}_t(\pi_t, \alpha_T)\}_{t=T+1},\ \{\boldsymbol{q}_t(1, \alpha_T)\}_{t>T+1}\big\}_{t=0,\ T=0}^{\infty\quad \overline{T}}$ solves the bargaining problem in (2);

2. $\big\{\{\boldsymbol{w}_t(\pi_t, \alpha_t)\}_{t\leq T},\ \{\boldsymbol{w}_t(\pi_t, \alpha_T)\}_{t=T+1},\ \{\boldsymbol{w}_t(1, \alpha_T)\}_{t>T+1}\big\}_{t=0,\ T=0}^{\infty\quad \overline{T}}$ solves buyers' maximization problem in (3);

3. Beliefs update according to Bayes' rule in (1);

4. Currency markets clear.

For the rest of the paper, we focus on symmetric equilibrium where at least one currency is valued and aggregate real balances are stationary after agents learn acceptance has stopped growing.[13]

[12] These sets take into account how quantities traded and real balances are different when acceptance is growing, in the period that agents realize acceptance stopped growing, and periods of known constant acceptance respectively.

[13] In a proof similar to one from Rocheteau and Wright (2013), we can prove existence of equilibrium. Email authors for details.

2.3 Steady State Analysis

We first analyze an economy in steady state with constant real balances, so acceptance α is known and constant and $\pi = 1$. Following LW, steady state allocations are tractable and well understood; this will allow us to study the economy outside steady state. We use upper bars to denote steady state prices. In steady state, the opportunity costs are proportional to currency growth rates, so $\bar{i}^m_\alpha = \frac{\bar{\phi}_\alpha}{\beta \bar{\phi}'_\alpha} - 1 = \frac{1+\gamma_m}{\beta} - 1$ and $\bar{i}^c_\alpha = \frac{\bar{\psi}_\alpha}{\beta \bar{\psi}'_\alpha} - 1 = \frac{1+\gamma_c}{\beta} - 1$. The buyer's maximization problem from (3) becomes

$$\max_{m',c'} \Big\{ -\bar{i}^m_\alpha \bar{\phi}'_\alpha m' - \bar{i}^c_\alpha \bar{\psi}'_\alpha c' + \lambda\theta \big[\alpha S(\bar{\phi}'_\alpha m' + \bar{\psi}'_\alpha c') + (1-\alpha) S(\bar{\phi}'_\alpha m')\big] \Big\}. \tag{4}$$

Using the definitions of opportunity costs and effective wealth and first order conditions, it is easy to show prices satisfy

$$\bar{\phi}_\alpha \geq \bar{\phi}'_\alpha \beta \big[1 + \alpha\lambda\ell(\bar{\varpi}'_\alpha + \bar{\upsilon}'_\alpha) + (1-\alpha)\lambda\ell(\bar{\varpi}'_\alpha)\big], \tag{5}$$

$$\bar{\psi}_\alpha \geq \bar{\psi}'_\alpha \beta \big[1 + \alpha\lambda\ell(\bar{\varpi}'_\alpha + \bar{\upsilon}'_\alpha)\big]. \tag{6}$$

If $m' > 0$ and $c' > 0$, each of the above conditions hold with equality. The steady state equilibrium prices of money and cryptocurrency only depend on currency growth rates, the discount factor, and acceptance rates.[14] Proposition 1 below establishes the conditions under which cryptocurrency and money will be held.

Proposition 1. *In steady state, there exists a stationary money-cryptocurrency equilibrium,*

[14]Because money and cryptocurrency are not perfect substitutes, we do not need to worry about exchange rate indeterminacy as in Kareken and Wallace (1981). See also Zhang (2014).

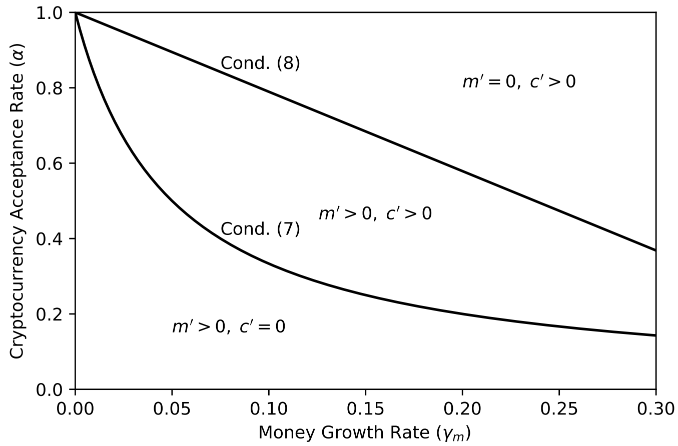

Figure 1: **Steady State Equilibrium Type:** Type of equilibrium in steady state based on currency growth rate γ_m and cryptocurrency acceptance rate α, with cryptocurrency acceptance rate $\gamma_c = 0$.

i.e. $m' > 0$ *and* $c' > 0$ *if and only if* [15]

$$1 + \gamma_c < \alpha(1 + \gamma_m) + (1 - \alpha)\beta, \tag{7}$$

$$\frac{\gamma_m - \gamma_c}{\beta} < \lambda(1 - \alpha)\frac{\theta}{1 - \theta}. \tag{8}$$

Proof. See Appendix A. □

Figure 1 shows how equilibrium type changes with currency growth rate γ_m and acceptance rate α, holding cryptocurrency growth rate constant at a fixed γ_c. In the state space above Condition (7), cryptocurrency is valued in equilibrium while below it is not. The intuition behind this condition is that in meetings with crypto sellers, which happen with probability α, money and cryptocurrency are substitutes, so cryptocurrency could have a currency growth rate as high as that of money and still be valued. However, in meetings with money type sellers, which happen with probability $1 - \alpha$, cryptocurrency has no liquidity value and thus needs to have a return equal to the rate of time preference β for it to be held, i.e. satisfy the Friedman rule. Because buyers are risk neutral in the CM, the

[15] Although there will still exist non-monetary equilibria where one or both currencies are not valued in the model.

maximum growth rate that cryptocurrency can have and still be valued is the probability weighted average of these two types of meetings. As the money growth rate or acceptance rate increases, cryptocurrency becomes more valuable relative to money, hence it is more likely to be held.

In the state space below Condition (8), money is valued in equilibrium while above it is not. The intuition behind this condition is that the difference in opportunity cost between money and cryptocurrency (LHS) needs to be smaller than the benefit of trading with a money seller (RHS). The term $\frac{\theta}{1-\theta} = \ell(0)$ represents the liquidity value of a marginal unit of effective wealth when the buyer has none. As the money growth rate or acceptance rate increases, money becomes less valuable relative to cryptocurrency, hence it is less likely to be held. Figure 1 is for a fixed γ_c. From Conditions (7) and (8), it is clear that increasing γ_c will shift Condition (7) to the right and rotate Condition (8) clockwise. This means cryptocurrency is less likely to be valued.

We are also interested in characterizing total welfare in steady state. If we add up the value functions of all buyers and sellers starting from each DM, we get total welfare each period is

$$\mathcal{W}_\alpha = \lambda\Big\{\alpha S(\overline{\omega}_\alpha + \overline{\nu}_\alpha) + (1-\alpha)S(\overline{\omega}_\alpha)\Big\}, \tag{9}$$

which is the expected trade amount in the DM. Note that trade surplus $S(\cdot)$ is increasing and concave in effective wealth, which has important implications for comparative statics as we will see below.

In Table 1, we show how quantities traded, prices, and total welfare change with currency growth rates and cryptocurrency acceptance rate.[16] The first row examines how these factors are affected by increases in money growth rate γ_m. Increased growth rate means the currency loses value faster, so the price of money $\bar{\phi}'$ decreases and trade with money sellers q^m does too. Buyers substitute for lost money holdings by using more cryptocurrency, driving up the price $\bar{\psi}'$ and leaving trade with crypto sellers q^c unchanged. Lower trade with money sellers means

[16] Calculations are provided in Appendix B.

χ	$\frac{\partial q^c}{\partial \chi}$	$\frac{\partial q^m}{\partial \chi}$	$\frac{\partial \bar{\phi}'}{\partial \chi}$	$\frac{\partial \bar{\psi}'}{\partial \chi}$	$\frac{\partial \mathcal{W}}{\partial \chi}$
γ^m	0	-	-	+	-
γ^c	-	+	+	-	?
α	+	-	-	+	?

Table 1: **Effects of Steady State Parameter Changes**

less trade overall, so welfare decreases. The second row shows that increasing cryptocurrency growth rate γ_c has the opposite effect on quantities and prices, but now trade with crypto sellers decreases because money loses value faster (which must be true if cryptocurrency is valued). The third row shows the effects of increasing cryptocurrency acceptance α. This increases the liquidity value of cryptocurrency, so its price rises and buyers demand less money and more cryptocurrency. Lower money demand lowers its price and quantity traded with money sellers, but buyers still hold more liquidity overall and quantity traded with crypto sellers increases. Both cryptocurrency growth rate and acceptance rate have ambiguous effects on total welfare. On one hand, as cryptocurrency becomes more valuable as a currency (as γ_c decreases or α increases), trade with crypto sellers increases which increases welfare. On the other hand, it leads to less valuable money and trade with money sellers decreases, which lowers total welfare, especially because the total surplus function is concave. In Appendix B, we show that welfare effects of more valuable cryptocurrency tend to be negative when effective wealth is already high and cryptocurrency acceptance is very low, meaning the lower trade with money sellers is very costly.

From analyzing steady state, we have shown how currency acceptance mainly depends on currency stock growth rates and acceptance rate. As today's cryptocurrency acceptance increases, the price of cryptocurrency increases, and it's more likely to be valued. Because money and cryptocurrency are partial substitutes, money prices fall with rising cryptocurrency prices. This can have ambiguous effects on welfare, making cryptocurrency more valuable can have negative affects on total welfare. In the next section, we use this steady state knowledge to analyze our dynamic problem where acceptance is increasing over time.

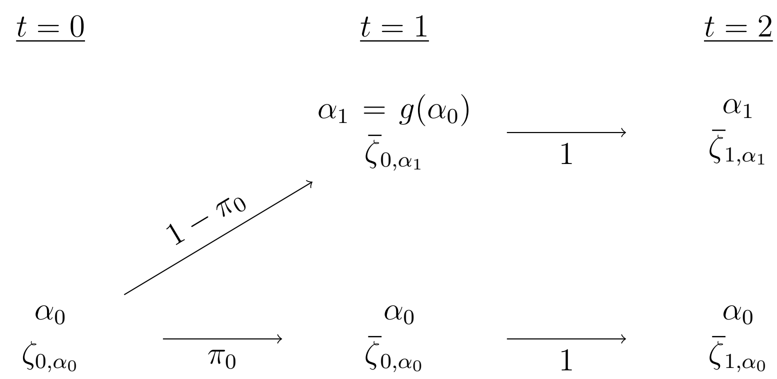

Figure 2: **Dynamics of Acceptance Rates $\boldsymbol{\alpha}$ and Real Balances $\boldsymbol{\zeta} \in \{\boldsymbol{\phi M}, \boldsymbol{\psi C}\}$**

2.4 Dynamics

Now we want to examine how the model behaves outside of steady state. We can characterize the equilibrium by starting in the steady state with the highest acceptance rate and working backwards. Figure 2 demonstrates a simple example with $T \in \{0, 1\}$, so $\overline{T} = 1$ is the largest possible termination date. In the DM of $t = 0$, α is revealed to be α_0 with probability π_0 or $\alpha_1 = g(\alpha_0) > \alpha_0$ with probability $1 - \pi_0$. If $\alpha = \alpha_0$ then $T = 0$, the world reached the low acceptance steady state, and $\alpha = \alpha_0$ forever. If $\alpha = \alpha_1$, then $T = 1$, growth will stop after $t = 1$, and α_1 stays constant forever. We call this the high acceptance steady state.

We want to solve the ex ante price in period 0, where the future is known to be either a high or a low acceptance steady state. The opportunity cost of money and cryptocurrency are $i^m_{\alpha_0} = \frac{\phi_{0,\alpha_0}}{\beta[\pi_0 \bar{\phi}_{1,\alpha_0} + (1-\pi_0)\bar{\phi}_{1,\alpha_1}]} - 1$ and $i^c_{\alpha_0} = \frac{\psi_{0,\alpha_0}}{\beta[\pi_0 \bar{\psi}_{1,\alpha_0} + (1-\pi_0)\bar{\psi}_{1,\alpha_1}]} - 1$. Buyers solve the following problem

$$\begin{aligned} \max_{m_1, c_1} \Big\{ & - i^m_{\alpha_0} \big[\pi_0 \bar{\phi}_{1,\alpha_0} + (1-\pi_0)\bar{\phi}_{1,\alpha_1}\big] m_1 - i^c_{\alpha_0} \big[\pi_0 \bar{\psi}_{1,\alpha_0} + (1-\pi_0)\bar{\psi}_{1,\alpha_1}\big] c_1 \\ & + \lambda\theta \Big[\pi_0 \big[\alpha_0 S(\omega_{1,\alpha_0} + \upsilon_{1,\alpha_0}) + (1-\alpha_0) S(\omega_{1,\alpha_0})\big] \\ & \quad + (1-\pi_0)\big[\alpha_1 S(\omega_{1,\alpha_1} + \upsilon_{1,\alpha_1}) + (1-\alpha_1) S(\omega_{1,\alpha_1})\big] \Big] \Big\}, \end{aligned} \tag{10}$$

which implies prices satisfy

$$\phi_{0,\alpha_0} \geq \beta\Big\{\pi_0\bar{\phi}_{1,\alpha_0}\big[1+\alpha_0\lambda\ell(\omega_{1,\alpha_0}+\upsilon_{1,\alpha_0})+(1-\alpha_0)\lambda\ell(\omega_{1,\alpha_0})\big]$$
$$+(1-\pi_0)\bar{\phi}_{1,\alpha_1}\big[1+\alpha_1\lambda\ell(\omega_{1,\alpha_1}+\upsilon_{1,\alpha_1})+(1-\alpha_1)\lambda\ell(\omega_{1,\alpha_1})\big]\Big\} \tag{11}$$
$$\psi_{0,\alpha_0} \geq \beta\Big\{\pi_0\bar{\psi}_{1,\alpha_0}\big[1+\alpha_0\lambda\ell(\omega_{1,\alpha_0}+\upsilon_{1,\alpha_0})\big]+(1-\pi_0)\bar{\psi}_{1,\alpha_1}\big[1+\alpha_1\lambda\ell(\omega_{1,\alpha_1}+\upsilon_{1,\alpha_1})\big]\Big\}, \tag{12}$$

with equality if each currency is held in equilibrium. Prices are probability weighted averages of low and high acceptance steady state prices. Because cryptocurrency prices increase in acceptance ($\bar{\psi}_{1,\alpha_0} < \bar{\psi}_{1,\alpha_1}$), ex ante price at period 0 will be higher than the period 0 steady state even though acceptance is the same, $\psi_{0,\alpha_0} > \bar{\psi}_{0,\alpha_0}$. Because money prices decrease in acceptance ($\bar{\phi}_{1,\alpha_0} > \bar{\phi}_{1,\alpha_1}$), ex ante prices will be lower when acceptance is growing compared to the period 0 steady state, $\phi_{0,\alpha_0} < \bar{\phi}_{0,\alpha_0}$. Cryptocurrency price in the low acceptance steady state is positive if Condition (7) is satisfied, otherwise it is zero. Even if this price is zero, prices may still be positive today if agents are optimistic enough about reaching the high acceptance steady state. Our model can generate short term positive prices of cryptocurrency even if fundamentals never allow for long-run viability of the currency.

For economies with longer potential growth $\overline{T} > 2$, we solve the model using backwards induction. Because period $\overline{T}+1$ is always a steady state, we can solve for prices in period $\overline{T}$. Then in period $\overline{T}-1$, tomorrow's acceptance either grows to this state or stays constant, either way prices are known, so this period's prices can also be solved for. This continues to the initial date, giving us the potential path of prices. In the next subsection, we show a numerical example for a longer period of potential acceptance growth.

2.5 Numerical Example

We now show some numerical examples. For functional forms, we choose $u(q) = \frac{(q+b)^{1-\eta}-b^{1-\eta}}{1-\eta}$ and $h(q) = q$, where b is a small number to ensure a solution to the bargaining problem. We take $g(\alpha) = (1-\delta)\alpha + \delta\bar{\alpha}$, where $\bar{\alpha} \leq 1$ is some exogenous upper bound on the measure of crypto sellers; acceptance grows by a percentage δ of the distance between the maximum possible acceptance rate and current acceptance rate each period. We assume the buyers' prior is such that $\pi_t = \pi \; \forall \; t < \overline{T}$, so each period before $\overline{T}$ buyers have constant belief that growth will stop.[17]

The growth path of real balances of cryptocurrency and money are shown in the left and right panels of Figure 3.[18] The graphs compare real balances if acceptance grows to real balances if acceptance stops growing and the economy enters steady state with constant acceptance. The first x-axis measures time t, which moves linearly, while the second measures acceptance α, which grows according to $g(\cdot)$. For example, in the left panel, the solid yellow line represents the real balances of cryptocurrency while acceptance is still growing. As acceptance grows each period, real balances also grow. If growth stops, real balances crash to the steady state level of the dashed black line where they will stay forever. The dotted red line shows one potential path when $T = 20$. Before $t = 20$, real balances of cryptocurrency grow following the solid yellow line. At $t = 21$, agents realize acceptance has stopped growing, and the real balance immediately crashes to the steady state level where it stays forever.

For cryptocurrency, real balances during the growth phase are higher than the corresponding steady state. As we saw in the previous subsection, the possibility that prices will be higher if acceptance is still growing tomorrow lowers the cryptocurrency inflation rate buyers face and makes cryptocurrency more attractive today. Even though cryptocurrency will not be viable in steady state until period 16, where α is high enough to satisfy Condition 7 of Proposition 1, cryptocurrency will still have positive prices starting in period 0. Once acceptance stops growing, real balances crash to steady state value and stay constant thereafter. The increased demand for cryptocurrency decreases the demand for money, so

[17] Parameters used are $b = 0.00001,\ \beta = 0.95,\ M_0 = 1,\ C_0 = 1,\ \gamma_m = 0.02,\ \gamma_c = 0,\ \eta = 0.3,\ \lambda = 0.5,\ \theta = 0.5\ \alpha_0 = 0,\ \alpha_g = 0.1,\ \pi = 0.1,\ \delta = 0.1,$ and $\overline{T} = 50$.

[18] We show total real balances rather than prices because inflation decreases prices over time, and we want to differentiate the change of prices due to inflation from the change of prices due to more cryptocurrency acceptance.

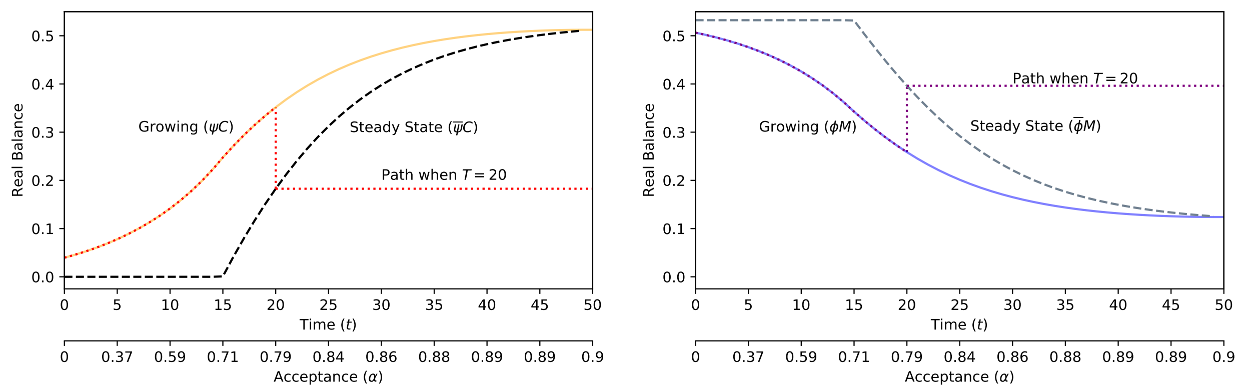

Figure 3: **Real Balances:** Total real balances of cryptocurrency (left) and money (right) while acceptance is still growing versus when it is not growing (steady state) and an example path when $T = 20$. The x-axis shows both time and acceptance rate.

its price is lower than it would be in steady state. When growth stops, the real balances of money increase.

To see how beliefs affect prices, Figure 4 shows how different priors $\pi \in \{0, .1, .2\}$ that acceptance growth will stop each period change price curves. We again show cryptocurrency real balances on the left and money real balances on the right. When $\pi = 0$, buyers believe acceptance will always grow until $\overline{T}$, so they believe there is no risk in holding cryptocurrency. As a result, real balances of cryptocurrency are much higher and real balances of money are much lower. By contrast, when $\pi = .2$, buyers are much more skeptical about the future of cryptocurrency but it still has positive prices for low acceptance. In general, cryptocurrency real balances monotonically decrease with π while money real balances monotonically increase. When all buyers are more optimistic, cryptocurrency prices increase; however, as we will show in the next section, when only some buyers are more optimistic, price changes are ambiguous.

In the above example, the long-run price of cryptocurrency is positive only if acceptance stops growing after period 16, which happens less than 20% of the time. Even though cryptocurrency's long-run viability is in doubt, people still value it because it facilitates transactions next period. The potential for future growth lowers expected cryptocurrency inflation, making in cheaper to use in transactions and worthwhile to hold. Through this

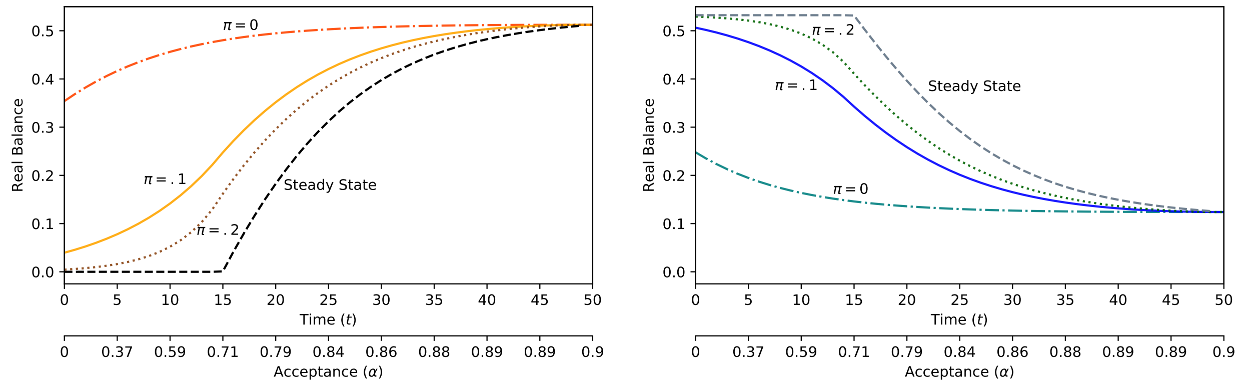

Figure 4: **Real Balances Changing Beliefs:** Total real balances of cryptocurrency (left) and money (right) for different priors $\pi = \{0, .1, .2\}$.

mechanism, our model justifies holding cryptocurrency for transaction purposes even when few sellers accept cryptocurrency today. But because acceptance growth may stop, our model can also generate crashes in prices, both to zero and positive values.

How does the presence of cryptocurrency affect welfare? Figure 5 shows total welfare from our numerical examples change with cryptocurrency acceptance both for steady state with the dashed black line and while acceptance is growing with the solid yellow line. As we saw in comparative statics there are two competing forces affecting steady state welfare. On one hand, cryptocurrency holds its value better than money does, so effective wealth in crypto meetings increase in acceptance. On the other hand, cryptocurrency holdings replace money, so effective wealth in money meetings is lower. Initial increases in acceptance make cryptocurrency valuable enough to hold even if many sellers do not accept it, lowering total welfare. When enough sellers accept cryptocurrency, the increase in total real balances are enough to increase total welfare. Adding dynamics means that cryptocurrency is worth more and that buyers hold cryptocurrency even when very few sellers accept it. It also means the value of buyer's cryptocurrency is uncertain; usually it is worth slightly more than what they purchased it for but it may be worth much less or worthless. When acceptance is low, these effects combine to make welfare much lower than the steady state case; as acceptance increases, they make welfare a little bit higher.

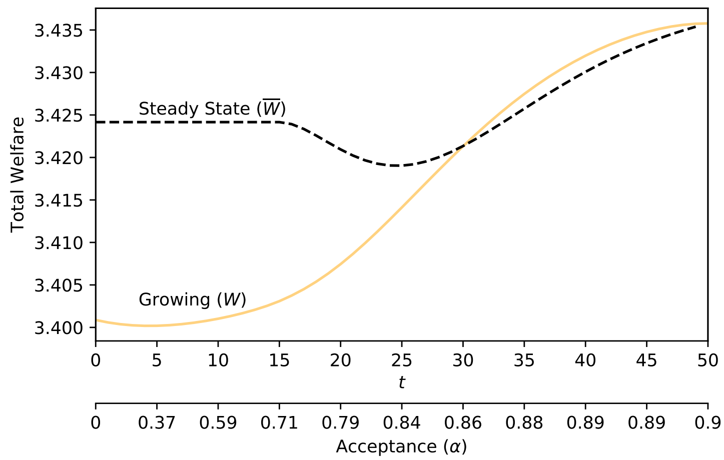

Figure 5: **Total Welfare:** Total welfare starting in the DM of each period while acceptance is still growing versus when it is not growing (steady state). The x-axis shows both time and acceptance rate.

Overall, our numerical examples show how important dynamics can be: the potential for higher future acceptance means cryptocurrency can have positive price even when acceptance is very low. Potential for acceptance growth means cryptocurrency prices are above steady state while money prices are lower, and these effects grow stronger as buyers become more optimistic. When acceptance stops growing, prices immediately crash to steady state levels. The presence of dynamics has uncertain effects on total welfare; it initially is below steady state levels but increases above steady state as acceptance grows. In the next section, we will extend the model with heterogeneous beliefs and show how this changes the model's predictions.

3 Hodlers Extension

Our baseline model assumed all buyers have the same beliefs, but some early adopters started valuing cryptocurrencies before others. We want to see how these optimistic agents affect currency values and normal buyers' portfolios. To do so, we model an extension where buyers' beliefs about acceptance growth are heterogeneous. We add optimistic buyers called hodlers (see Footnote 7) who have measure $1-\mu$ and *a priori* believe that α will always grow until $\overline{T}$.[19] We denote hodlers' asset holdings and beliefs with a tilde. A measure μ of buyers are normal buyers who behave exactly as in the baseline model. Despite their different priors, once acceptance stops growing and the economy enters steady state, hodlers and buyers behave identically, as they now share the same beliefs. As for sellers' beliefs, because information arrives at the beginning of the DM, sellers learn no new information between receiving currency payments in the DM and selling back these currencies in the next CM. As such, our results hold as long as sellers are (weakly) less optimistic about cryptocurrency than hodlers. In Appendix C, we define a sequential monetary equilibrium similar to the one in Definition 2.1.

We want to characterize how prices are determined with the addition of hodlers. To compare with the baseline model, we use the same scenario as the dynamics in Section 2.4 and study the problem in period 0. The normal buyers' problem and first order conditions are the exact same as in the baseline model (see Equations (11) and (12)), so we focus here on the hodlers' problem. In steady state, α is known, so the hodlers value function is given by Equation (4). Because hodlers have the same steady state values as normal buyers, we can solve their out-of-steady-state problem in a similar fashion. Before learning the termination date T, hodlers believe α is always growing, so their prior is $\tilde{\pi}_0 = 0$. After solving the hodlers' problem, prices satisfy:

$$\phi_{0,\alpha_0} \geq \bar{\phi}_{1,\alpha_1}\beta\big[1 + \alpha_1\lambda\ell(\tilde{\omega}_{1,\alpha_1} + \tilde{\upsilon}_{1,\alpha_1}) + (1-\alpha_1)\lambda\ell(\tilde{\omega}_{1,\alpha_1})\big], \tag{13}$$

$$\psi_{0,\alpha_0} \geq \bar{\psi}_{1,\alpha_1}\beta\big[1 + \alpha_1\lambda\ell(\tilde{\omega}_{1,\alpha_1})\big]. \tag{14}$$

If hodlers value money and cryptocurrency then each condition holds with equality. Notice that the above conditions are special cases of Equations (11) and (12) with $\pi_0 = 0$. Intuitively, hodlers are more optimistic about cryptocurrency's future and thus they value cryptocurrency more than regular buyers and hold more cryptocurrency. The opposite is

[19] Formally, their prior is $\tilde{f}(\overline{T}) = 1$.

true for money. Because prices are determined by marginal values, which decrease in effective wealth, it is still possible for both types to hold both currencies. In fact, the effect of adding holders on marginal valuation of both currencies is ambiguous, so it is possible for cryptocurrency prices to fall and money prices to rise from the baseline case. We will discuss this in detail in the next section.

Equilibrium in this simple example is characterized by market clearing conditions $\mu m + (1-\mu)\tilde{m} = M$ and $\mu c + (1-\mu)\tilde{c} = C$ plus the first order conditions from (11)-(12) and (13)-(14). For more general cases with more periods of potential growth, we can follow the same logic from Section 2.4 and use backwards induction to find each potential price sequence. We show a numerical example of such a case in the next section.

3.1 Numerical Example

To see how the model works, we use the same parameters as Footnote 17 but now add hodlers to the economy. We compare results with different fractions of normal buyers $\mu = \{.5, .99, 1\}$.[20] Real balances are shown in Figure 6 and asset holdings are shown in Figure 7. First we look at the left panel of Figure 6 concerning cryptocurrency real balances. The internal margin of hodler population works much like the internal margin of beliefs π from Figure 4; as we add hodlers to the economy, the average beliefs about acceptance growth become more optimistic and cryptocurrency price rises. The external margin of adding hodlers from the baseline is less certain. When acceptance is low, cryptocurrency prices are higher in the presence of hodlers because they are more optimistic about the future of cryptocurrency, which is especially important in determining prices when α and liquidity value are small. But as acceptance grows, adding only a few hodlers $\mu = .99$, leads to a lower price compared to the baseline case. When there are few hodlers, they hold lots of cryptocurrency, which lowers the liquidity premium of a marginal unit of cryptocurrency, lowering their marginal demand compared to the baseline case. Normal buyers have lower

[20]Note that $\mu = 1$ is equivalent to the economy in Figure 3. Also note that an economy with only hodlers is equivalent to one where $\pi = 0$ in Figure 4.

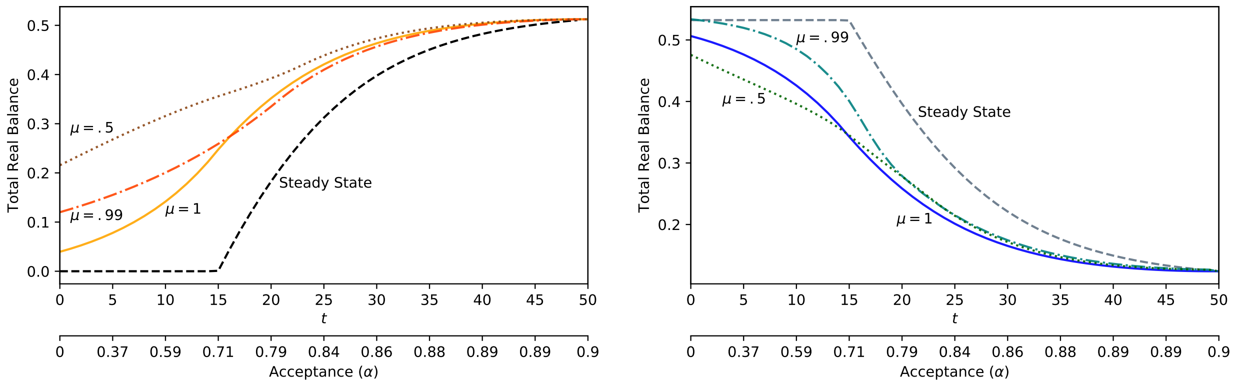

Figure 6: **Real Balances with Changing Hodler Population:** Total real balances of cryptocurrency (left) and money (right) for different populations of buyers and hodlers. For each line, μ is the measure of normal buyers and $1-\mu$ is the measure of hodlers.

cryptocurrency real balances, but, as we will see in Figure 7 below, they get more liquidity from money which is valued in all meetings and has less competition from hodlers. Therefore, marginal demand for cryptocurrency decreases, lowering the price. As the proportion of hodlers increases, each individual hodler holds less cryptocurrency so the marginal unit has a higher liquidity premium and hodler competition drives up prices. With enough hodlers, prices are always above the baseline case.

We now look at the right panel concerning money real balances. Again, the effect of adding hodlers is ambiguous. For low levels of acceptance, money prices are higher for smaller populations of hodlers and lower for larger populations of hodlers. There are two forces affecting money prices compared to the baseline case. The first is that normal buyers are crowed out of the cryptocurrency market (see left panel of Figure 7) and they replace this loss of liquidity by demanding more money. The second is that hodlers demand less money but because cryptocurrency still has low acceptance, they still have moderate demand for money (see right panel of Figure 7). With only a few hodlers in the economy, the first force dominates. With a lot of hodlers, the second force dominates.

As acceptance increases, the size of these two forces changes. While normal buyers are crowded out of the cryptocurrency market, their lost cryptocurrency liquidity increases with acceptance and their relative demand for money increases. When they begin to hold

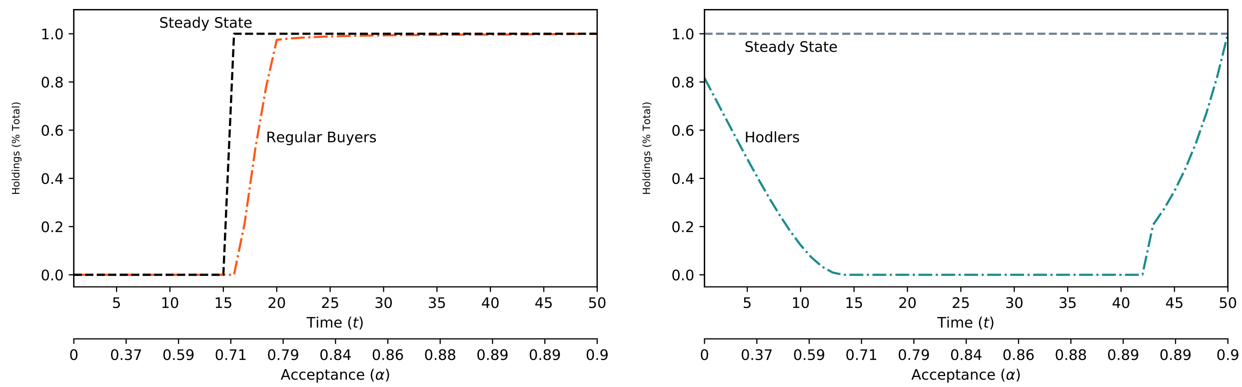

Figure 7: **Currency Holdings for $\mu = .99$:** Cryptocurrency holdings for normal buyers (left) and money holding for hodlers (right) while acceptance is still growing versus when it is not growing (steady state) for $\mu = .99$. normal buyer holdings are for a measure .1 of buyers for ease of comparison.

cryptocurrency, they still hold fewer real balances, either because cryptocurrency is worth less or because their holdings are very low. In either case, their demand for money is higher than the baseline case. As acceptance grows, cryptocurrency becomes more liquid and hodlers demand less money. But eventually normal buyers' beliefs converge to hodlers', so their allocations converge as well and hodlers hold more money. In our numerical example, the first effect dominates as acceptance increases, so money prices stay above the baseline case both with few and with many hodlers. But for other specifications, for example in an economy with only hodlers as is the case when $\pi = 0$ in Figure 4 above, money prices can be lower than the baseline case.

Our model shows that the interaction between agents with heterogeneous beliefs has a non-obvious effect on prices. While hodlers are more optimistic about the future of cryptocurrency and will hold more cryptocurrency than buyers, they may have negative effects on cryptocurrency prices. This occurs because their cryptocurrency holdings can be so large that their marginal demand is lower. Adding hodlers can increase money prices both in the short- and long-run. This extension shows how optimistic agents may crowd regular people out of cryptocurrency when acceptance is low and drive up cryptocurrency prices, but if optimist's predictions come true, normal people will start to use it as well when many sellers accept it. While this has clearly not happened yet globally, cryptocurrencies have been taken up in countries like Venezuela where money inflation is extreme.

4 Conclusion

Our model studies the transaction motive for holding cryptocurrency and relates uncertain acceptance of cryptocurrency to both high values of cryptocurrency today even when seller acceptance is low and crashes in price even when seller acceptance remains unchanged. Our crashes can occur even with rational agents with correct beliefs and can result in both positive and zero long-run prices. We show that in steady state, cryptocurrency prices rise with money growth rate and acceptance rate and fall with cryptocurrency growth rate. We use our steady state results to backwards induct prices when acceptance is growing and show the potential for growth increases the value of cryptocurrency today. This can result in positive short-run prices even when acceptance is low and positive prices are unlikely in the long-run. Additionally, we extend the baseline with heterogeneous beliefs about future seller acceptance. Optimistic agents value cryptocurrency more but surprisingly their presence may lead to lower cryptocurrency prices.

The main contribution of our paper is the focus on how future acceptance affects the value of currencies today. While our paper is inspired by cryptocurrency, it can be applied to a broader context of competing currencies, such as dollarization, where a widely accepted domestic currency faces competition from a more stable foreign currency. Our model is highly stylized. We assume one cycle of boom and bust; there is no potential for acceptance to grow once it has stopped. In reality, acceptance growth may start and stop, and prices will reflect this. Cryptocurrencies are not like many other fiat currencies, in that their supply and transaction fees depend on mining games, something we abstract from here.

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Appendices

A Proof of Proposition 1

Proof. First, we take $m' > 0$ as given, so the first order condition from Equation (4) for money binds. If Condition (7) does not hold then $1+\gamma_c \geq \alpha(1+\gamma_m)+(1-\alpha)\beta$. Rearranging gives

$$\frac{1+\gamma_c}{\beta} - 1 \geq \alpha\left[\frac{1+\gamma_m}{\beta} - 1\right].$$

From the definition of interest rates and binding money first order condition, this implies $i^c \geq \lambda\alpha\big[\alpha\ell(\omega'_\alpha + \upsilon'_\alpha) + (1-\alpha)\ell(\omega'_\alpha)\big]$, where, for ease of notation, we suppress all bars representing steady state.

For contradiction, suppose cryptocurrency holdings are positive $c' > 0$ and the first order condition for cryptocurrency binds. This implies $\omega'_\alpha + \upsilon'_\alpha > \omega'_\alpha$ which means $\ell(\omega'_\alpha + \upsilon'_\alpha) < \ell(\omega'_\alpha)$. By the first order condition of cryptocurrency

$$i^c = \lambda\alpha\ell(\omega'_\alpha + \upsilon'_\alpha) \geq \lambda\alpha\big[\alpha\ell(\omega'_\alpha + \upsilon'_\alpha) + (1-\alpha)\ell(\omega'_\alpha)\big],$$

which would imply $\ell(\omega'_\alpha + \upsilon'_\alpha) \geq \ell(\omega'_\alpha)$, a contradiction. This implies $c' = 0$ if (7) does not hold.

Secondly, we take $c' > 0$ as given, so the first order condition of Equation (4) for cryptocurrency binds. If Condition (8) does not hold then $\frac{\gamma_m - \gamma_c}{\beta} \geq \lambda(1-\alpha)\frac{\theta}{1-\theta}$. From the definition of interest rates, binding cryptocurrency first order condition, and assumptions about $u(\cdot)$ and $c(\cdot)$, we can show $i^m \geq \lambda\big[\alpha\ell(\omega'_\alpha + \upsilon'_\alpha) + (1-\alpha)\frac{\theta}{1-\theta}\big] = \lambda\big[\alpha\ell(\omega'_\alpha + \upsilon'_\alpha) + (1-\alpha)\ell(0)\big]$.

For contradiction, suppose money holdings are positive $m' > 0$ and the first order condition for money binds. This implies $\omega'_\alpha > 0$ which means $\ell(\omega'_\alpha) < \ell(0)$. By the first order condition of money

$$i^m = \lambda\big[\alpha\ell(\omega'_\alpha + \upsilon'_\alpha) + (1-\alpha)\ell(\omega'_\alpha)\big] \geq \lambda\big[\alpha\ell(\omega'_\alpha + \upsilon'_\alpha) + (1-\alpha)\ell(0)\big],$$

which would imply $\ell(\omega'_\alpha) \geq \ell(0)$, a contradiction. This implies $m' = 0$ if (8) does not hold. $\square$

B Steady State Comparative Statics

For ease of notation, in the following section we suppress all bars representing steady state and denote values from the previous period with a -1 subscript, i.e i_{-1}. Assume m, $c > 0$ so that prices are positive and first order conditions bind. Let $\lambda^c = \lambda\alpha$ and $\lambda^m = \lambda(1-\alpha)$. First we find comparative statics for currency growth rate changes. Note that in steady state, $i^j_{-1} = \frac{1+\gamma_j}{\beta} - 1 \; \forall \; j \in \{m, c\}$. Our first order conditions from Equation (4) are

$$i^m_{-1} = \lambda^c\ell(\omega_\alpha + \upsilon_\alpha) + \lambda^m\ell(\omega_\alpha), \tag{15}$$

$$i^c_{-1} = \lambda^c\ell(\omega_\alpha + \upsilon_\alpha). \tag{16}$$

We start with the money growth rate. Taking partial derivatives of (15)-(16) with respect to γ_m

$$1 = \lambda^c\ell'(\omega_\alpha + \upsilon_\alpha)\frac{\partial[\omega_\alpha + \upsilon_\alpha]}{\partial\gamma_m} + \lambda^m\ell'(\omega_\alpha)\frac{\partial\omega_\alpha}{\partial\gamma_m},$$
$$0 = \lambda^c\ell'(\omega_\alpha + \upsilon_\alpha)\frac{\partial[\omega_\alpha + \upsilon_\alpha]}{\partial\gamma_m}.$$

Using Cramer's rule, we can rewrite the above equations as

$$\begin{bmatrix} \lambda^c\ell'(\omega_\alpha + \upsilon_\alpha) & \lambda^m\ell'(\omega_\alpha) \\ \lambda^c\ell'(\omega_\alpha + \upsilon_\alpha) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial[\omega_\alpha + \upsilon_\alpha]}{\partial\gamma_m} \\ \frac{\partial\omega_\alpha}{\partial\gamma_m} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The determinant of this matrix is $\Delta \equiv -\lambda^c \ell'(\omega_\alpha + \upsilon_\alpha)\lambda^m \ell'(\omega_\alpha) < 0$. Then $\frac{\partial[\omega_\alpha+\upsilon_\alpha]}{\partial\gamma_m} = \frac{0}{\Delta} = 0$, and $\frac{\partial\omega_\alpha}{\partial\gamma_m} = -\frac{\lambda^c\ell'(\omega_\alpha+\upsilon_\alpha)}{\Delta} < 0$. Because $q(.)$ is monotonically increasing, this shows $\frac{\partial q^c}{\partial\gamma_m} = 0$ and $\frac{\partial q^m}{\partial\gamma_m} < 0$.

Using the definition of $z(\cdot)$, we find $\phi' = \frac{z[q(\omega_\alpha)]}{m'}$ and $\psi' = \frac{z[q(\omega_\alpha+\upsilon_\alpha)]-z[q(\omega_\alpha)]}{c'}$ and using the implicit function theorem we find $q'(w) = \frac{1}{z'[q(w)]}$. From this, we can derive $\frac{\partial\phi'}{\partial\gamma_m} = \frac{z'[q(\omega_\alpha)]q'(\omega_\alpha)}{m'}\frac{\partial\omega_\alpha}{\partial\gamma_m} = \frac{-\lambda^c\ell'(\omega_\alpha+\upsilon_\alpha)}{m'\Delta} < 0$ and $\frac{\partial\psi'}{\partial\gamma_m} = \frac{z'[q(\omega_\alpha+\upsilon_\alpha)]q'(\omega_\alpha+\upsilon_\alpha)}{c'}\frac{\partial[\omega_\alpha+\upsilon_\alpha]}{\partial\gamma_m} - \frac{z'[q(\omega_\alpha)]q'(\omega_\alpha)}{c'}\frac{\partial\omega_\alpha}{\partial\gamma_m} = 0 + \frac{\lambda^c\ell'(\omega_\alpha+\upsilon_\alpha)}{c'\Delta} > 0$.

For total welfare, taking the derivative of (9) with respect to γ_m gives

$$\begin{aligned}\frac{\partial\mathcal{W}}{\partial\gamma_m} &= \frac{\lambda^c_\alpha}{\theta}\ell(\omega_\alpha+\nu_\alpha)\frac{\partial[\omega_\alpha+\nu_\alpha]}{\partial\gamma_m} + \frac{\lambda^m_\alpha}{\theta}\ell(\omega_\alpha)\frac{\partial\omega_\alpha}{\partial\gamma_m} \\ &= \frac{\ell(\omega_\alpha)}{\theta\ell'(\omega_\alpha)} \leq 0,\end{aligned} \tag{17}$$

with strict inequality if $\gamma_m > \beta - 1$, i.e. money does not satisfy the Friedman rule. Unsurprisingly, people are better off with a low money growth rate.

Now we look at the cryptocurrency growth rate. Taking partial derivatives of (15)-(16) with respect to γ_c

$$\begin{aligned}0 &= \lambda^c\ell'(\omega_\alpha+\upsilon_\alpha)\frac{\partial[\omega_\alpha+\upsilon_\alpha]}{\partial\gamma_c} + \lambda^m\ell'(\omega_\alpha)\frac{\partial\omega_\alpha}{\partial\gamma_c}, \\ 1 &= \lambda^c\ell'(\omega_\alpha+\upsilon_\alpha)\frac{\partial[\omega_\alpha+\upsilon_\alpha]}{\partial\gamma_c}.\end{aligned}$$

Using Cramer's rule, we can rewrite the above equations as

$$\begin{bmatrix}\lambda^c\ell'(\omega_\alpha+\upsilon_\alpha) & \lambda^m\ell'(\omega_\alpha) \\ \lambda^c\ell'(\omega_\alpha+\upsilon_\alpha) & 0\end{bmatrix}\begin{bmatrix}\frac{\partial[\omega_\alpha+\upsilon_\alpha]}{\partial\gamma_c} \\ \frac{\partial\omega_\alpha}{\partial\gamma_c}\end{bmatrix} = \begin{bmatrix}0 \\ 1\end{bmatrix}.$$

Notice the determinant Δ is the same as above. Then $\frac{\partial[\omega_\alpha+\upsilon_\alpha]}{\partial\gamma_c} = -\frac{\lambda^m\ell'(\omega_\alpha)}{\Delta} < 0$ and $\frac{\partial\omega_\alpha}{\partial\gamma_c} = \frac{\lambda^c\ell'(\omega_\alpha+\upsilon_\alpha)}{\Delta} > 0$. This shows $\frac{\partial q^c}{\partial\gamma_c} < 0$ and $\frac{\partial q^m}{\partial\gamma_c} > 0$. Using the definition of $z(\cdot)$ and the implicit function theorem as above, we can show $\frac{\partial\phi'}{\partial\gamma_c} = \frac{z'[q(\omega_\alpha)]q'(\omega_\alpha)}{m'}\frac{\partial\omega_\alpha}{\partial\gamma_c} = \frac{\lambda^c\ell'(\omega_\alpha+\upsilon_\alpha)}{m'\Delta} > 0$ and

$\frac{\partial \phi'}{\partial \gamma_c} = \frac{z'[q(\omega_\alpha+\upsilon_\alpha)]q'(\omega_\alpha+\upsilon_\alpha)}{c'} \frac{\partial[\omega_\alpha+\upsilon_\alpha]}{\partial \gamma_c} - \frac{z'[q(\omega_\alpha)]q'(\omega_\alpha)}{c'} \frac{\partial \omega_\alpha}{\partial \gamma_c} = \frac{-\lambda^m \ell'(\omega_\alpha) - \lambda^c \ell'(\omega_\alpha+\upsilon_\alpha)}{c'\Delta} < 0.$

For total welfare, taking the derivative of (9) with respect to γ_c gives

$$\begin{aligned}
\frac{\partial \mathcal{W}}{\partial \gamma_c} &= \frac{\lambda^c_\alpha}{\theta} \ell(\omega_\alpha + \nu_\alpha) \frac{\partial[\omega_\alpha + \nu_\alpha]}{\partial \gamma_c} + \frac{\lambda^m_\alpha}{\theta} \ell(\omega_\alpha) \frac{\partial \omega_\alpha}{\partial \gamma_c} \\
&= \frac{\lambda^m_\alpha \lambda^c_\alpha}{\theta \Delta} [\ell(\omega_\alpha)\ell'(\omega_\alpha + \nu_\alpha) - \ell(\omega_\alpha + \nu_\alpha)\ell'(\omega_\alpha)] \qquad (18) \\
&= \frac{1}{\theta} \left[\frac{\ell(\omega_\alpha + \nu_\alpha)}{\ell'(\omega_\alpha + \nu_\alpha)} - \frac{\ell(\omega_\alpha)}{\ell'(\omega_\alpha)} \right], \qquad (19)
\end{aligned}$$

which is positive if and only if $\frac{\ell'(\omega_\alpha)}{\ell(\omega_\alpha)} > \frac{\ell'(\omega_\alpha+\nu_\alpha)}{\ell(\omega_\alpha+\nu_\alpha)}$. While the sign of this comparison is ambiguous, there is generally a level of wealth for which increasing the cryptocurrency growth rate increases welfare, as shown in Lemma B.1 below

Lemma B.1. Let $P(w) = \frac{\ell'(w)}{\ell(w)}$. There exists a $\hat{w}$ such that for all $\omega_\alpha > \hat{w}$, $P'(\omega_\alpha) < 0$ and $\frac{\partial \mathcal{W}}{\partial \gamma_c} > 0$.

Proof. Note that (throughout suppressing the reliance of u, h, and z on $q(w)$)

$$P'(w) \equiv \frac{\ell(w)\ell''(w) - \ell'(w)^2}{\ell(w)^2}$$

where

$$l'(w) \equiv \theta \frac{u''h' - u'h''}{(z')^3} < 0$$

and

$$l''(w) \equiv \theta \frac{z'(u'''h' - u'h''') - 3z''(u''h' - u'h'')}{(z')^5}$$

which in general has an ambiguous sign. Let $q(w^*) = q^*$. As $w \to w^*$, $\ell(w) \to 0$ by definition, but in general $\ell'(w) \nrightarrow 0$ and $\ell''(w) \nrightarrow 0$ (additionally, neither diverges to infinity or negative infinity). So there exists a $\hat{w}$ such that if $w > \hat{w}$, $\frac{\partial P(w)}{\partial w} < 0$, $\frac{\ell'(\omega_\alpha)}{\ell(\omega_\alpha)} > \frac{\ell'(\omega_\alpha+\nu_\alpha)}{\ell(\omega_\alpha+\nu_\alpha)}$, and $\frac{\partial \mathcal{W}}{\partial \gamma_c} > 0$, which we wanted to show. □

Lemma B.1 says that when effective wealth is high enough and cryptocurrency acceptance is low enough, then the losses from buyers holding less effective wealth in crypto meetings is offset by them holding more effective wealth in money meetings and total welfare increases in cryptocurrency growth rate.

Now we find the comparative statics for acceptance rate of cryptocurrency α. Solving (15)-(16) for α, we obtain $\alpha = \frac{i^m_{-1} - \lambda\ell(\omega_\alpha)}{\lambda[\ell(\omega_\alpha+\upsilon_\alpha)-\ell(\omega_\alpha)]}$ and $\alpha = \frac{i^c_{-1}}{\lambda\ell(\omega_\alpha+\upsilon_\alpha)}$. Using Cramer's rule, we can rewrite these as

$$\begin{bmatrix} \frac{-[i^m_{-1}-\lambda\ell(\omega_\alpha)]\ell'(\omega_\alpha+\upsilon_\alpha)}{\lambda[\ell(\omega_\alpha+\upsilon_\alpha)-\ell(\omega_\alpha)]^2} & \frac{\ell'(\omega_\alpha)[i^m_{-1}-\lambda\ell(\omega_\alpha+\upsilon_\alpha)]}{\lambda[\ell(\omega_\alpha+\upsilon_\alpha)-\ell(\omega_\alpha)]^2} \\ \frac{-i^c_{-1}\ell'(\omega_\alpha+\upsilon_\alpha)}{\lambda\ell(\omega_\alpha+\upsilon_\alpha)^2} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial[\omega_\alpha+\upsilon_\alpha]}{\partial\alpha} \\ \frac{\partial\omega_\alpha}{\partial\alpha} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

The determinant of this matrix is $\Delta_\alpha = \frac{i^c_{-1}\ell'(\omega_\alpha+\upsilon_\alpha)\ell'(\omega_\alpha)[i^m_{-1}-\lambda\ell(\omega_\alpha+\upsilon_\alpha)]}{\lambda^2\ell(\omega_\alpha+\upsilon_\alpha)^2[\ell(\omega_\alpha+\upsilon_\alpha)-\ell(\omega_\alpha)]^2} > 0$. Because $i^m_{-1} - \lambda\ell(\omega_\alpha+\upsilon_\alpha) = \lambda(1-\alpha)[\ell(\omega_\alpha)-\ell(\omega_\alpha+\upsilon_\alpha)] > 0$ and $i^m_{-1}-\lambda\ell(\omega_\alpha) = \lambda\alpha[\ell(\omega_\alpha+\upsilon_\alpha)-\ell(\omega_\alpha)] < 0$ we know $\frac{\partial[\omega_\alpha+\upsilon_\alpha]}{\partial\alpha} = \frac{-\ell'(\omega_\alpha)[i^m_{-1}-\lambda\ell(\omega_\alpha+\upsilon_\alpha)]}{\Delta_\alpha\lambda[\ell(\omega_\alpha+\upsilon_\alpha)-\ell(\omega_\alpha)]^2} > 0$ and $\frac{\partial\omega_\alpha}{\partial\alpha} = \ell'(\omega_\alpha+\upsilon_\alpha)\frac{i^c_{-1}[\ell(\omega_\alpha+\upsilon_\alpha)-\ell(\omega_\alpha)]^2-[i^m_{-1}-\lambda\ell(\omega_\alpha)]\ell(\omega_\alpha+\upsilon_\alpha)^2}{\Delta_\alpha\lambda\ell(\omega_\alpha+\upsilon_\alpha)^2[\ell(\omega_\alpha+\upsilon_\alpha)-\ell(\omega_\alpha)]^2} < 0$. This shows $\frac{\partial q^c}{\partial\alpha} > 0$ and $\frac{\partial q^m}{\partial\alpha} < 0$. Using the definition of $z(\cdot)$ and the implicit function theorem as above, we can show $\frac{\partial\phi'}{\partial\alpha} < 0$ and $\frac{\partial\psi'}{\partial\alpha} > 0$.

For total welfare, taking the derivative of (9) with respect to α gives

$$\begin{aligned} \frac{\partial\mathcal{W}}{\partial\alpha} =& \lambda\left[S(\omega_\alpha+\nu_\alpha) - S(\omega_\alpha)\right] + \frac{\lambda^c_\alpha}{\theta}\ell(\omega_\alpha+\nu_\alpha)\frac{\partial[\omega_\alpha+\nu_\alpha]}{\partial\alpha} + \frac{\lambda^m_\alpha}{\theta}\ell(\omega_\alpha)\frac{\partial\omega_\alpha}{\partial\alpha} \\ =& \lambda\left[S(\omega_\alpha+\nu_\alpha) - S(\omega_\alpha)\right] + \frac{1}{\theta\Delta_\alpha[\ell(\omega_\alpha+\nu_\alpha)-\ell(\omega_\alpha)]^2}\times \\ & \left\{\ell(\omega_\alpha)\ell'(\omega_\alpha+\nu_\alpha)(1-\alpha)\frac{i^c_{-1}[\ell(\omega_\alpha+\nu_\alpha)-\ell(\omega_\alpha)]^2 - [i^m_{-1}-\lambda\ell(\omega_\alpha)]\ell'(\omega_\alpha+\nu_\alpha)^2}{\ell(\omega_\alpha+\nu_\alpha)^2}\right. \\ & \left. - \ell(\omega_\alpha+\nu_\alpha)\ell'(\omega_\alpha)\alpha[i^m_{-1}-\lambda\ell(\omega_\alpha+\nu_\alpha)]\right\}, \end{aligned} \tag{20}$$

the sign of which is generally ambiguous.

C Equilibrium Definition with Hodlers

Definition C.1. Let $\boldsymbol{q} = (q^{cs}, q^{ms})$, $\tilde{\boldsymbol{q}} = (\tilde{q}^{cs}, \tilde{q}^{ms})$, $\boldsymbol{w} = (\omega, \upsilon)$, and $\tilde{\boldsymbol{w}} = (\tilde{\omega}, \tilde{\upsilon})$. Given initial prior $\{\hat{\pi}_t\}_{t=1}^{\overline{T}}$ of normal buyers and prior $\tilde{\pi}_{\overline{T}} = 1$ of hodlers, we define a sequential monetary equilibrium as a list of normal buyers' quantities traded $\left\{\{\boldsymbol{q}_t(\pi_t,\alpha_t)\}_{t\leq T},\ \{\boldsymbol{q}_t(\pi_t,\alpha_T)\}_{t=T+1},\ \{\boldsymbol{q}_t(1,\alpha_T)\}_{t>T+1}\right\}_{t=0,\ T=0}^{\infty\ \ \overline{T}}$, hodlers' quantity traded $\left\{\{\tilde{\boldsymbol{q}}_t(0,\alpha_t)\}_{t\leq T},\ \{\tilde{\boldsymbol{q}}_t(0,\alpha_T)\}_{t=T+1},\ \{\tilde{\boldsymbol{q}}_t(1,\alpha_T)\}_{t>T+1}\right\}_{t=0,\ T=0}^{\infty\ \ \overline{T}}$, normal buyers' real balances $\left\{\{\boldsymbol{w}_t(\pi_t,\alpha_t)\}_{t\leq T},\ \{\boldsymbol{w}_t(\pi_t,\alpha_T)\}_{t=T+1},\ \{\boldsymbol{w}_t(1,\alpha_T)\}_{t>T+1}\right\}_{t=0,\ T=0}^{\infty\ \ \overline{T}}$, and hodlers' real balances $\left\{\{\tilde{\boldsymbol{w}}_t(0,\alpha_t)\}_{t\leq T},\ \{\tilde{\boldsymbol{w}}_t(0,\alpha_T)\}_{t=T+1},\ \{\tilde{\boldsymbol{w}}_t(1,\alpha_T)\}_{t>T+1}\right\}_{t=0,\ T=0}^{\infty\ \ \overline{T}}$ such that

1. Normal buyers' and hodlers' quantity traded solve the bargaining problem in (2);
2. Normal buyers' and hodlers' real balances solve buyers' maximization problem in (3);
3. Normal buyers' and hodlers' beliefs update according to Bayes' rule in (1);
4. Currency markets clear.

Like with Definition 2.1, we can prove equilibrium exits, email authors for details.

D Endogenizing α

We now let sellers choose whether to accept cryptocurrency or not. At the end of each CM, after buyers have chosen currency holdings, the cost of accepting is revealed and sellers can choose to pay a one-time cost to accept cryptocurrency forever. Each seller now has a type $\rho \in [\rho_{min}, \rho_{max}]$ and the one-time cost $\kappa_t(\rho)$ depends on type and period t. We assume that $\kappa_t(\rho)$ is observable by all buyers and sellers and evolves over time according to

$$\kappa_t(\rho) = \begin{cases} \rho g(\kappa_{t-1}) & \text{if } t < T \\ \rho\kappa_{t-1} & \text{if } t \geq T. \end{cases}$$

Where $T \in \{0, 1, ...\overline{T}\}$ is an unknown period and $g(\cdot)$ is a known function such that κ is decreasing over time ($g'(\cdot) < 1$), but is decreasing slowly enough such that if a seller would want to accept cryptocurrency tomorrow at today's cost, they will also want to accept today. The measure of sellers accepting cryptocurrency is α.

Sellers discover that costs stopped decreasing in period T when they reach the end of the CM of period $T+1$ and learn κ did not decrease. Agents have a common prior over the steady state date $F(T)$ with pdf $f(t)$. We use π_t to denote agents' beliefs about κ staying constant conditional on κ's history. If $t > T$, agents know that cost has stopped decreasing and $\pi_t = 1$. If $t \leq T$, agents know that cost decreased last period but are unsure whether it will decrease this period, so

$$\pi_t \equiv \mathbb{P}(\kappa_{t+1} = \kappa_t \,|\, \kappa_t < \kappa_{t-1}) = \mathbb{P}(t = T \,|\, t \leq T) = \frac{f(t)}{1 - F(t-1)}.$$

Because time is discrete, beliefs are not continuous. Define agents' prior as $F(T) = \{\hat{\pi}_0, \hat{\pi}_1, ..., \hat{\pi}_{\overline{T}}\}$ where $\hat{\pi}_t \equiv \mathbb{P}(t = T)$. Then

$$\pi_t = \begin{cases} \frac{\hat{\pi}_t}{1 - \sum\limits_{\tau < t} \hat{\pi}_\tau} & \text{if } t \leq T \\ 1 & \text{if } t > T. \end{cases} \tag{21}$$

We assume all agents have the same beliefs.

Intuitively, if more other sellers are accepting cryptocurrency, then the benefit of accepting cryptocurrency increases. As in Lester et al. (2012), this can lead to multiple equilibria with different levels of coordination of acceptance. We focus on equilibria where seller's level of coordination is constant, for example one where sellers always choose the highest level of coordination. Because of this, buyers solve the same problem as the exogenous α case for any given potential paths of α and we can find prices and currency holdings of buyers. As such, our model with exogenous acceptance can have the same outcome as a model with endogenous acceptance.

We now show how different levels of coordination can lead to different acceptance growth paths. Define $R(\rho, \alpha_{t+1}, t)$ as seller type ρ's expected net benefit of accepting cryptocurrency over only accepting money while cost is still decreasing and α_{t+1} other sellers will accept cryptocurrency next DM. Each period, sellers who do not yet accept cryptocurrency will do so if $R(\rho, \alpha_{t+1}, t) - \rho\kappa_t \geq 0$. The benefit of accepting cryptocurrency today is the expected benefit from accepting money and cryptocurrency for the future

$$(1-\theta)\lambda \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\mathbb{E}\left[S(\phi_{\tau,\alpha_\tau} m_\tau + \psi_{\tau,\alpha_\tau} c_\tau)\right]. \tag{22}$$

The benefit of not accepting today is tomorrow's benefit of only accepting money plus the ability to choose to accept cryptocurrency tomorrow based on another observation of κ. If κ stops decreasing and the seller would still want to accept cryptocurrency next period, then she will always want to accept cryptocurrency today (if κ not decreasing too quickly, which we assumed). If κ decreases tomorrow and the seller would still not want to accept cryptocurrency, then they will never want to accept it today. We focus on the in between sellers who would want to accept tomorrow if κ decreases but not if it stops decreasing. Their value of not accepting today is

$$(1-\theta)\lambda\beta S(\phi_{t+1,\alpha_{t+1}} m_{t+1}) + (1-\theta)\lambda \sum_{\tau=t+2}^{\infty} \beta^{\tau-t}\mathbb{E}\left[S(\phi_{\tau,\alpha_{t+1}} m_\tau)\right]$$
$$+ (1-\pi_t)\beta\left[R(\rho,\alpha_{t+2},t+1) - \rho\kappa_{t+1}\right]. \tag{23}$$

Taking (22) minus (23), the benefit of accepting cryptocurrency today is

$$\begin{aligned} R(\rho,\alpha_{t+1},t) =& (1-\theta)\lambda\beta\left\{S(\phi_{t+1,\alpha_{t+1}} m_{t+1} + \psi_{t+1,\alpha_{t+1}} c_{t+1}) - S(\phi_{t+1,\alpha_{t+1}} m_{t+1})\right\} \\ &+ (1-\theta)\lambda \sum_{\tau=t+2}^{\infty} \beta^{\tau-t}\left\{\mathbb{E}\left[S(\phi_{\tau,\alpha_\tau} m_\tau + \psi_{\tau,\alpha_\tau} c_\tau) - S(\phi_{\tau,\alpha_{t+1}} m_\tau)\right]\right\} \\ &- (1-\pi_t)\beta\left[R(\rho,\alpha_{t+2},t+1) - \rho\kappa_{t+1}\right]. \end{aligned} \tag{24}$$

It is clear that $\frac{\partial[R(\rho,\alpha_{t+1},t)-\rho\kappa]}{\partial\rho} < 0$, sellers with lower ρ are more likely to start accepting cryptocurrency. Let $\sim$ denote two terms have the same sign and note the partial with respect to α_{t+1} is

$$\frac{\partial\left[R(\rho,\alpha_{t+1},t) - \rho\kappa\right]}{\partial\alpha_{t+1}} \sim S'(\omega_{\alpha_{t+1}} + \upsilon_{\alpha_{t+1}})\frac{\partial[\omega_{\alpha_{t+1}} + \upsilon_{\alpha_{t+1}}]}{\alpha_{t+1}} - S'(\omega_{\alpha_{t+1}})\frac{\partial\omega_{\alpha_{t+1}}}{\partial\alpha_{t+1}}. \tag{25}$$

From our comparative statics, we argue $S'(\cdot) > 0$, $\frac{\partial[\omega_{\alpha_{t+1}} + \upsilon_{\alpha_{t+1}}]}{\alpha_{t+1}} > 0$, and $\frac{\partial \omega_{\alpha_{t+1}}}{\partial \alpha_{t+1}} < 0$, so $\frac{\partial[R(\rho, \alpha_{t+1}, t) - \rho\kappa]}{\partial \alpha_{t+1}} > 0$. This shows that higher cryptocurrency acceptance makes adopting it more profitable, leading to the potential for multiple equilibria.